CBCS SCHEME

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15MATDIP31

Third Semester B.E. Degree Examination, Jan./Feb. 2021
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: ***Answer FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Find the real and imaginary parts of $\frac{2+i}{3-i}$ and express in the form of x + iy. **(05 Marks)**

b. Reduce $1 - \cos\alpha + j\sin\alpha$ to the modulus amplitude form $[r(\cos\theta + i\sin\theta)]$ by finding r and θ. **(06 Marks)**

c. If $\vec{a} = 4i + 3j + k$ and $\vec{b} = 2i - j + 2k$ find the unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$. Hence show that $\sin\theta = \frac{\sqrt{185}}{3\sqrt{26}}$ where 'θ' is angle between $\vec{a}$ and $\vec{b}$. **(05 Marks)**

OR

2 a. Find the modulus and amplitude of $\frac{3+i}{1+i}$. **(05 Marks)**

b. Find 'a' such that the vectors $2i - j + k$, $i + 2j - 3k$ and $3i + aj + 5k$ are coplanar. **(06 Marks)**

c. Show that for any three vectors $\bar{a}, \bar{b}, \bar{c}$ $[\bar{b}\times\bar{c}, \bar{c}\times\bar{a}, \bar{a}\times\bar{b}] = [\bar{a}, \bar{b}, \bar{c}]^2$. **(05 Marks)**

Module-2

3 a. Find the n^{th} derivative of sin (5x) cos (2x). **(05 Marks)**

b. If y = a cos (log x) + b sin (log x) prove that $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(06 Marks)**

c. If $u = \sin^{-1}\frac{x+y}{\sqrt{x}-\sqrt{y}}$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$. **(05 Marks)**

OR

4 a. Expand $e^{\sin x}$ by Maclaurin's series upto the term containing x^4. **(05 Marks)**

b. Give $u - \sin\left(\frac{x}{y}\right)$ $x = e^t$ $y = t^2$ find $\frac{du}{dt}$ as a function of t. **(06 Marks)**

c. If $x = r\cos\theta$, $y = r\sin\theta$ find $\frac{\partial(x,y)}{\partial(r,\theta)}$ and $\frac{\partial(r,\theta)}{\partial(x,y)}$. **(05 Marks)**

Module-3

5 a. State reduction formula for $\int_0^{\pi/2}\sin^n x\,dx$ and evaluate $\int_0^{\pi/2}\sin^9 x\,dx$. **(05 Marks)**

b. Evaluate $\int_0^{\infty}\frac{dx}{(1+x^2)^{7/2}}$. **(06 Marks)**

c. Evaluate : $\int_0^1\int_0^2\int_0^2 x^2yz\,dx\,dy\,dz$. **(05 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

OR

6 a. Evaluate : $\int_0^{\pi} \sin^4 x \ \cos^6 x \, dx$. **(05 Marks)**

b. Evaluate : $\int_0^5 \int_0^{x^2} y(x^2 + y^2) dx \, dy$. **(06 Marks)**

c. Evaluate : $\int_0^1 \int_0^2 \int_1^2 x^3 y^2 z^3 \, dx \, dy \, dz$. **(05 Marks)**

Module-4

7 a. A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 3$ where t is the time. Find the velocity and acceleration at time $t = 1$. **(05 Marks)**

b. Find the unit normal vector to the surface $xy^3z^2 = 4$ at the point $(-1, -1, 2)$. **(06 Marks)**

c. What is solenoid vector field? Demonstrate that vector $\overline{F}$ given by $\overline{F} = 3y^2z^3 i + 8x^2 \sin(z) j + (x + y) k$ is solenoidal. **(05 Marks)**

OR

8 a. Find div F and Curl F if $\overline{F} = (3x^2 - 3yz)i + (3y^2 - 3xz)j + (3z^2 - 3xy)k$. **(05 Marks)**

b. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. **(06 Marks)**

c. Show that the fluid motion $\vec{V} = (y + z)i + (z + x)j + (x + y)k$ is irrotational. **(05 Marks)**

Module-5

9 Find the solution of :

a. $(x^2 + 2e^x)dx + (\cos y - y^2)dy = 0$. **(05 Marks)**

b. $\dfrac{dy}{dx} = \dfrac{y/x}{1 + y/x}$. **(06 Marks)**

c. $(x^2 - ay)dx + (y^2 - ax)dy = 0$. **(05 Marks)**

OR

10 a. Find the solution of :

$\dfrac{dy}{dx} = \dfrac{x^3}{y^3}$. **(05 Marks)**

b. $(x^2y^3 + \sin x)dx + (x^3y^2 + \cos y)dy = 0$. **(06 Marks)**

c. $\cos y \dfrac{dy}{dx} + \sin y = 1$. **(06Marks)**

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